Citation: Haury, David L., (2002), What is problem solving? In L. Milbourne (Ed.), The connected teacher's companion to problem solving in science and mathematics. Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education (pp. 1-31).

CHAPTER 1

What is Problem Solving?

Are you wondering if this is a trick question? Problem solving has long been such a familiar part of our school vocabulary that the question may seem trivial or simply rhetorical. Not so: it is a serious and important question, and how you answer the question will determine the relevance and value of this guide for you. There are many definitions of problem solving. You may immediately think of mathematics classes and the seemingly endless number of story or word problems that you solved during your years in school. Perhaps you once learned a set of procedures or skills to follow in stepwise fashion to solve example after example of nonmathematical problems. We want to be clear at the outset that we are not using "problem solving" in this guide to denote such activities. While it is true that most of us typically associate the solving of problems in school with mathematics, or we think of problem solving as a stepwise procedure to follow, problem solving in a broader view refers to something much more fundamental to human endeavor. Problem solving is, in fact, one of the mental habits that sets humans apart from other living beings. When confronted with a new situation or a perceived obstacle to accomplishing some goal, we typically do not just stop and choose another goal or course of action. We think about the situation, plan a strategy for getting beyond the obstacle to accomplish our goal, and attempt to follow the plan, making changes or adjustments as needed to solve whatever problem confronts us. This is human nature. Indeed, most of us solve several problems each day, from figuring out how to pay the monthly bills when financial resources are limited to retrieving keys that are locked inside a car. Problems vary widely in the challenges they present and the techniques required for solving them. Some problems require the application of mathematical procedures, while others require special knowledge or skills.

In Benchmarks for Scientific Literacy (p. 282, Project 2061, 1993), the point is made that we live in "a world in which problems abound-in the home, in the workplace, in the community, on the planet." A major purpose of schooling, then, is "preparing students to become effective problem solvers, alone and in concert with others." This is where learning in science and mathematics are essentially searches for solutions to problems, from highly theoretical problems related to our understanding of the natural world, to immediately practical problems associated with daily living. The habits of mind associated with doing science or mathematics have broad application in our technological world, and students who learn to nurture these habits of mind increase their potential for solving the myriad problems that confront us. In this guide we present some basic steps and skills associated with solving particular kinds of problems in science and mathematics, but the larger goal is to bring into view the general habits of mind associated with recognizing and solving problems involving broader concepts or issues related to science or mathematics.

So, how do we define problem solving in this guide? First we need a working definition of "problem" so we can bring focus to our instruction on problem solving. The definition must be broad enough to fit the wide variety of problems we all face in our daily lives, yet precise enough to guide instruction. Here is the definition of "problems" that we are using in this guide:

Situations confronting a person or a group that must be resolved, but for which no solution or paths to solutions are readily apparent.

These are problems. Solving problems will take different forms and paths in different settings and contexts, but the end point will always be the same: problem solved!

Though our definition of "problems" may seem simple, or even simplistic, behind the simplicity is a minefield of challenges for teachers. First, a situation is transformed into a problem only when an individual or group regards the situation as a problem condition. So first there is an acknowledgement or "acceptance" issue; a potential problem is perceived, and resolving the situation is considered important.

Once a potential problem is perceived, an individual or group will likely try to resolve the situation by resorting to a variety of strategies that have typically worked in past situations of a similar nature. When these familiar strategies fail to resolve the situation, the situation becomes a problem. Attempts to resolve the situation using familiar responses do not work, so the solution to a recognized problem is not readily apparent. If this problem is important enough to an individual or group, a search for possible problem solving strategies will begin. That is where this guide comes in. The guide is not a compilation of problems, but it does provide a semi-structured way to approach problem solving in a variety of contexts. There is no single formula or problem solving recipe that will solve all problems, but there is a general approach that helps focus attention on the central phases of problem solving. There are also specific techniques that can be used in certain situations or contexts.

Are there different types of problem solving?

This question is a little trickier. Though an ability to recognize and solve problems is a characteristically human ability, and though several generalized models of problem solving have been proposed over the years, solving specific problems is a highly contextualized process. A variety of discrete skills and strategies (heuristics) have been described for a range of problem types, and a wide variety of instructional approaches have been advocated for developing specific skills. So, the answer to the question about whether there are different types of problem solving is, no and yes. There is a general sense in which one begins by recognizing and defining a problem, and then following a logical path of reasoning and action to solve the problem. The way problem solving procedures are often described, however, typically varies from subject area to subject area, and the skills, procedures or heuristics used to solve specific problems will largely be determined by the contexts of the problems. The dual nature of problem solving, being both a general rational process and a rich milieu of contextualized procedures and heuristics, has led to complexities in both describing and studying problem solving in action.

We have chosen to address the complexities of the problem solving duality by:

- Presenting some representative models of problem solving that have long been associated with science and mathematics.
- Introducing a generalized model to provide the structure for this guide.
- Describing various problem solving skills and heuristics in the context of our generalized mode.
- Providing examples of how various models have been used to structure teaching and learning of problem solving in science and mathematics.
- Identifying references, resources, and research findings that are associated with the various problem solving models and heuristics.
- Though we are attempting to bring together the rich instructional and research traditions to provide guidance in developing problem solving abilities among students, you have the task of translating these ideas into instructional practices that are consistent, coherent, and fruitful within your classrooms. We want to provide a functional framework for your task without erecting a rigid structure, and we want to introduce a wide range of specific procedures and skills without making problem solving seem like a random process of trying one heuristic after another.

We begin laying the foundation by introducing some of the prominent general models that have been proposed within science and mathematics courses. In the sections that follow, we introduce generalized models of mathematical problem solving, scientific problem solving, creative problem solving, and information problem solving. Creative problem solving and information problem solving have been included because of their centrality to ongoing innovation in our culture, a culture that values invention and ongoing advances in science and technology.

Domains or Types of Problems to Solve

Mathematical problem solving

Instructional programs should enable all students to

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

(NCTM Principles and Standards, 2000)

We begin with mathematical problem solving because it has received the most attention for the longest period of time by educators and researchers, and because problem solving is explicitly mentioned within national education standards as being fundamental to literacy in mathematics. Indeed, Halmos (1980) characterized problem solving as the heart of mathematics, "that the mathematician's main reason for existence is to solve problems." This has long been a shared view among mathematics educators. The National Council of Supervisors of Mathematics (NCSM) ranked problem solving first among ten essential proficiencies over 20 years ago (NCSM, 1977), and the National Council of Teachers of Mathematics (NCTM) soon put problem solving at the top of its agenda (NCTM, 1980) where it remains to this day as first among the process standards for learning in mathematics (NCTM, 2000).

Traditionally, the problems in mathematical problem solving are usually expressed in narrative form, that is, word problems. These problems have been introduced to students after they learned basic skills. Students are expected to converse the circumstances to equivalent computation or equations, and to solve them by using the arithmetic or algebraic skills they have learned. Unfortunately, solving problems that are designed to practice a set of particular formulas or computational skills does not provide students with enough opportunities to be good problem solvers. It is because mathematical instruction might be focused on either practicing specific computational skills to given problems or being familiar with the specific case of the problem. Then teacher's role is likely to transmit an algorithm or information about how to solve the given problem. Eventually, it is not pretty much expected that all our students become good problem solvers when they face to figure out problems in their life in which the problems are not similar to those learned in mathematics classes.

Here, a problem does not simply mean a "word" problem. A problem for problem solving is not designed to improve for a particular mathematical skill, either. A "good" problem is what stimulates overall mathematical learning. The contexts of the problems can vary from familiar experiences involving students' lives or the school day to applications involving other disciplines. We want our students to get chances to solidify and extend what they know so that mathematical learning is stimulated well. Problem solving allows students to be engaged in a mathematical task for which the solution method is not known in advance.

Actually, problem solving is an integral part of all mathematics learning. We anticipate students to learn not simply how to solve problems but how to "do" mathematics via problem solving. Students' ability to create, conjecture, explore, test, and verify, all of which mean doing mathematics, can be developed throughout mathematical problem solving tasks (NCTM, 2000). To become a good problem solver is not easy. A significant amount of effort would be required in order for students to have frequent opportunities to formulate, cope with, and solve mathematical problems. But we expect all students to be good problem solver so that they can get great advantages in everyday life as well as mathematics class. Through the process to find a solution, draw on their knowledge, and reflect on their thinking, students will often develop new mathematical understandings (NCTM, 2000). By learning problem solving in mathematics, a student is expected to attain ways of mathematical thinking, to get interests and curiosity to new situations, and to be confident in solving unfamiliar problems.

Mathematics teachers play an important role in improving students' problem solving abilities (Lester et al., 1994). Not all students can become effective problem solvers on their own, but we believe that, with the help of a confident and capable teacher, all students can develop into effective problem solvers. Moreover, as a teacher becomes more comfortable at teaching problem solving, the teacher becomes more confident to help students understand a process that once seemed too complicated to figure it out (Susan, 2000). Then, classroom environment will be created and maintained to support students problem-solving procedures. As a consequence, students will develop confidence in their abilities and a willingness to engage in and explore problems.

In order to improve students' problem solving abilities, mathematics teachers need to learn appropriate teaching style for problem solving. It means that to teach as an act of transmitting information to passive students does not guarantee students' improvement in their problem solving abilities. It is necessary for teachers to regard teaching as an act of helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics (Lester et al., 1994). This change requires a correspondingly fundamental change in the teacher's role in the classroom. Rather than serve as the ultimate authority and dispenser of knowledge, the teacher variously plays the roles of guide, coach, question asker, and co-solver of problems.

First of all, mathematics teachers need to have a deep insight to anticipate mathematical ideas that can be brought out by working on a problem. Then teachers can decide wisely if a particular problem will help to further mathematical goal for the class. Teachers need to build instruction that help students find the mathematics in their worlds and experiences and by encouraging students to persist with interesting problems with appropriate level of difficulty (NCTM, 2000).

Secondly, mathematics teachers need to let students use proper strategies for solving problems. Using diagrams, looking for patterns, listing all possibilities, trying special values or cases, working backward, guessing and checking, creating an equivalent problem, and creating a simple problem are frequently used as strategies for developing students' problem solving abilities (Pólya, 1957). Here, students must become aware of these strategies as the need arises and as they are modeled during classroom activities. So, mathematics teachers needs to build instruction for students to experience a wider variety of problem solving strategies. Since no strategy is learned once and for all, students need to learn problem-solving strategies over time, and mathematics teacher s should give enough opportunities to students so that they apply several strategies to various contexts.

Thirdly, mathematics teachers need to help students constantly reflect on what they are doing (Lester, 1994). By letting students monitor and assess themselves constantly, and adjust their strategies, teachers can encourage students to solve problems (NCTM, 2000). Teachers play an important role in maintaining an environment in which students develop reflective skills, called "metacognition". Through questioning, teachers can help students get into the practice of checking their understanding as they go along. Teachers need to maintain an environment in which students learn to take responsibility for reflecting on their work and make the adjustments necessary when solving problems.

Mathematical problem solving has an important role in helping students develop creative and critical thinking. In addition, students' activities in mathematics classes that assist them in solving mathematical problems also help them become good problem solvers by learning mathematical knowledge heuristically. The heuristics used in solving problems differ from the algorithms taught in mathematics classrooms. An algorithm guarantees success if applied correctly and if the proper algorithm has been selected. Algorithms are task specific, whereas problem solving requires a more general approach (Krulik & Rudnick, 1995). Unlike an algorithm, heuristics cannot guarantee success. However, if students are taught to follow heuristics in every problem situation, they will be in a good position to successfully resolve the problems they face in life as well as in the mathematics classroom (Krulik & Rudnick, 1995). Students' successful experiences of managing their own knowledge can also assist them in solving mathematical problems well (Shoenfeld, 1985; Boaler, 1998). When problems are centered learning in mathematics, activities provide students enough chances to think critically, represent their own creative ideas, and communicate with their peers mathematically.

Resources

Following are selected resources related to mathematical problem solving. Resources include online web resources and resources indexed by the ERIC system.

Web Resources:

Similar resources can be found by using the Google search engine and search terms such as math OR mathematics OR mathematical "problem solving."

21st Century Problem Solving

http://www.hawaii.edu/suremath/home.html

Mathematics Through Problem Solving

http://www.mathgoodies.com/articles/problem_solving.shtm

Mathematics Problem Solving Scoring Guide

http://mh034.k12.sd.us/mathematics_problem_solving_scor.htm

Conceptual understanding: Scoring guide.

Teaching Values Through a Problem-Solving Approach to Mathematics

http://www.mathgoodies.com/articles/teaching_values.shtm

Mathematics Achievement through Problem Solving (MAPS): Teaching Mathematics for Elementary Teachers through Problem Solving

http://www.ed.gov/pubs/EPTW/eptw6/eptw6g.html

Problem Solving and Critical Thinking in Mathematics: A K-8 Professional Development and Interdisciplinary Curriculum Development Project

http://www.ncrel.org/sdrs/areas/issues/content/cntareas/math/ma4prob.htm

Project and product description.

Instructional Approaches to Teaching Problem Solving in Mathematics: Integrating Theories of Learning and Technology

http://www.mindymac.com/educ6100projects/TjonesProblem6100.htm

This paper outlines instructional approaches used in teaching the concept of mathematics problem solving and integrating theories of learning and technology.

ERIC Citations:

Brown, D. L., & Wheatley, Grayson H. (1997). Components of imagery and mathematical understanding. *Focus on Learning Problems in Mathematics,* 19 (1), 45-70.

Investigates students' use of imagery in their mathematical activities. Findings indicate that image forming is crucial in doing mathematics. A student who fails to construct an image in a problem-solving situation is severely limited in giving meaning to the situation. EJ 548 084.

Fernanadez, M. L. et al. (1994). Connecting research to teaching: problem solving: Managing it all. *Mathematics Teacher,* 87 (3), 95-199.

Discusses research involving instruction to help students develop their mental managerial processes or metacognition, the role of a framework for problem-solving activities, the teacher as model or moderator, problem solving involving groups or pairs, and weaving writing into problem solving. EJ 485 511.

McClintock, R. M (199?). The pyramid question: A problem-solving adventure. *Mathematics Teacher,* 90 (4), 262-268.

Presents a question designed to launch a discovery journey through conjecture, research, serendipitous encounters, proof, answers, and new questions. Reports some discoveries and suggests ways in which to incorporate this strategy into classrooms. Presents a geometry project that incorporates this problem-solving approach to mathematics. EJ 543 518.

Nunokawa, K. (1997). Data versus conjectures in mathematical problem solving. *Focus on Learning Problems in Mathematics,* 19 (1), 1-19.

Analyzes the problem-solving process and presents the dominance of the conjecture or theoretical aspect as a subtle relation between the data and the conjecture. Argues that conjectures are not necessarily subordinate to the data but play active roles in mathematical problem solving. Discusses the meaning of this dominance in the context of philosophy. EJ 548 082.

Rose, T. D., & Schuncke, G. M. (1997). Problem solving: The link between social studies and mathematics. *Clearing House,* 70 (3), 137-140.

Examines the problem-solving processes of social studies and mathematics and discusses their commonalities. Considers how those processes might be taught so that students will see that there is indeed a relationship between those two seemingly discrete disciplines. EJ 545 772.

Thorson, A. (Ed.). (1999). Inquiry and problem solving. In *ENC Focus,* 6 (2). Columbus, OH: Eisenhower National Clearinghouse, The Ohio State University.

This issue of ENC Focus focuses on the topic of inquiry and problem solving. Featured articles include: (1) "Inquiry in the Everyday World of Schools" (Ronald D. Anderson); (2) "In the Cascade Reservoir Restoration Project Students Tackle Real-World Problems" (Clint Kennedy with Advanced Biology Students from Cascade High School); (3) "Project Snoop Troop Cultivates a Community of Learners" (Julia Harris); (4) "Petals Around the Rose: Building Positive Attitudes about Problem Solving" (Marie Appleby); (5) "Roll with It: An Activity Integrating Mathematics and Science" (Bill Heinmiller); (6) "Inquiring Minds Find New Challenges in Mathematics Competitions, Contests, and Events" (Terese Herrera, Leah Poynter, and Judy Spicer); (7) "Walter Wick's Tricks Engage Young Minds, and Selecting Books? NSTA Provides Guidance" (Annette Thorson); (8) "20 Ways to Foster Creativity in Your Students" (Laura C. Mohr); (9) "'Students Questioning Students' Leads to Better Learning" (Judith Engel with Mathematics Students at The Bronx High School of Science); and (10) "Classroom Resources for Inquiry and Problem Solving" (Terese Herrera and Kimberly S. Roempler). Educational news, editorials, essays, classroom stories, and columns on topics of interest to classroom innovators are also included. ED 433 242.

Zambo, R. (1993). *Classroom practices in mathematical problem solving instruction: Arizona Elementary and Unified School Districts.* Survey results. Paper presented at the Annual Meeting of the School Science and Mathematics Association (Fresno, CA, October 14-16, 1994).

In spite of the continued focus on problem solving, American elementary and middle grades schoolchildren are perceived to be ineffective problem solvers. This paper reports the results of a survey of (n=744) kindergarten through eighth grade teachers and interviews and observations of a primary and an intermediate master teacher designed to answer the following questions: (1) What is the current nature of mathematical problem-solving instruction in Arizona K-8 classrooms, and (2) To what extent do reported classroom practices in problem-solving instruction reflect the recommendations of the National Council of Teachers of Mathematics (NCTM) and others? Survey results in the areas of instructional practices, word problem sources, student and self-assessments, problem-solving strategies, and beliefs are reported. Interview results include the topics of problem-solving instruction, definition of problem solving, use of textbooks, group work, assessment, calculators, and advice to other teachers. Three responses indicated possible misalignment with the NCTM Standards: (1) Use of manipulatives dropped off dramatically as grade level increased; (2) Teachers responded positively to students' needing to know the key word approach to problem solving; and (3) There was a discrepancy between the perceived usefulness of calculators and the actual use of calculators. ED 365 518.

Scientific Problem Solving

The need for improving science education to develop a scientifically literate generation leads educators and researchers to explore new possibilities in instruction and other areas of education. As an instructional method, problem solving has been an interesting topic for educators and researchers since the 1960s. In disciplines focusing on the outcome of a product or strategies, such as science and engineering, problem solving is the major application. In most cases, it is integrated with critical thinking and creativity. In education, learning scientific problem solving depends on the teacher, student and the content.

Over the last decade, one important often discussed issue in science education is to help students develop their problem-solving skills (Tolman & Hardy, 1999). There is extensive support for the importance of teaching inquiry processes and developing problem-solving skills in science education as evidenced by documents from national organizations) (e.g., American Association for the Advancement of Science [AAAS], 1989; National Council of Teachers of Mathematics [NCTM], 2000; and National Research Council [NRC], 1996). For example, *The National Science Education Standards* (National Research Council, 1996) puts great emphasis on problem solving and inquiry to nurture children's already existing curiosity by providing educational problems relevant to real-world. In many goal-oriented disciplines of products or strategies, such as science and engineering, problem solving is the major application in most cases integrated with critical thinking and creativity. As also stated in the "Habits of Mind" Benchmarks of Science Literacy: Project 2061, "there is a need for education to prepare students to make their way in the real world, a world in which problems abound—in the home, in the workplace, in the community, on the planet" (Project 2061). Science has an important contribution in the process of preparing students to become effective problem solvers. Benchmarks for science literacy identifies a list of skills necessary for effective problem solving: knowledge, skills and attitudes; computation and estimation; manipulation and observation; and communication and critical response skills.

Generally, what is problem solving? Krulik and Rudnick (1987) stated that problem solving is "the means by which an individual uses previously acquired knowledge, skills and understanding to satisfy the demands of an unfamiliar situation (p. 3). In other words, problem solving as a process approach differs from individual to individual based on their cognitive abilities and experiences. Success in problem solving can be maintained by providing cognitive behaviors. Different models are developed for this purpose by dividing the problem solving process into steps. Models will be explained in detail in the next section. Then, more specifically, what is scientific problem solving or the scientific method of problem solving? In science education, problem solving can be the application of science process skills that all children need. According to Tolman and Hardy (1999), there a little different lists of processes skills depending on source, but they summarized 10 appropriate skills for the elementary grades: observing, inferring, classifying, measuring, predicting, communicating, using space-time relationships, formulating hypotheses, identifying and controlling variables, and experimenting.

In educational settings, such as classrooms, most of the time students apply the known and available procedures to a well-defined problem to find an expected answer. In most cases very little learning occurs, and there is a lack of appreciation and understanding of the process of scientific problem solving. Students also experience difficulty in deciding on and choosing a matching procedure for a problem that is selected by them. As students gain more experience, their cognitive abilities increase.

Starting from early elementary grades, students are able to develop cognitive abilities of inquiry and capable of designing scientific investigations. Abstract presentations are meaningful in learning, and problem solving brings a broader sense to the meaning of the problem (Wheatley, 2001). Using real-world oriented problems helps students make the necessary connection with content they are being taught and the students have more experiences as their cognitive abilities increase (Wheatley, 1995).

Scientific problem solving is the process of solving problems by using those science process skills systematically. In general, the scientific method of problem solving includes the following steps:

1. Identification of the problem. A problem is any task that requires analysis and decision making to reason a solution. This step involves identifying and understanding the problem (Hauslein & Smith, 1995).
2. Consider current information and previous experience.
3. Hypothesis-developing a possible solution for the problem. This is one of the essential skills in inquiry. Hypothesis and predictions provides motivation and leads students to engage in higher-order critical thinking (Lavoie, 2001).
4. Testing the hypothesis. This step includes designing an experiment and set of procedures, collecting and analyzing data, and interpreting the results to determine whether or not the hypothesis is accurate.
5. Conclusion: the decision making step on the accuracy of hypothesis (ref: http://www.howe.k12.ok.us/~jimaskew/pmethod.htm).

How to improve problem solving skills-instructional strategies

Increasing students' ability to solve problems can be achieved by using a variety of instructional methods. Educational strategies for constructivist teaching method include conceptual change method, the learning cycle and generative learning model (Lavoie, 2001). All of these strategies aim to improve problem solving skills by focusing on the process rather than the result. Three types of knowledge should be known by teachers for an effective instruction to improve cognitive skills: conceptual knowledge, procedural knowledge and pedagogical knowledge in combination (Lavoie, 1995). Whimbey and Lochead (1999) identify two phases of teaching a skill; first demonstrating a skill to the students, then guiding and correcting them in practice. This two-phased teaching approach, unfortunately, does not work when it comes to problem solving because students can not observe the thinking process scientists go through in problem solving. Scientists experience the same difficulty in demonstrating their thinking process to students. Different instructional techniques can be applied to eliminate this barrier and increase students' ability to solve problems. Two examples of these techniques are think-aloud and pair problem solving. In most cases think-aloud procedure considered the base approach to eliminate the difficulty for both teaching and research purposes. This technique involves a person going through the process loud and describing all the steps and mental work during problem solving. Thus, students can understand the thinking process of the person or scientist. There are certain steps that problem solver needs to follow during the thinking aloud process. Hartman lists these steps as:

1. Translate your thoughts (e.g., ideas and images) into words and say them aloud.
2. Verbalize aloud all the steps that you go through when solving problems. Do not censor. No thought or step is too small, easy, obvious, or unimportant to verbalize.
3. Verbalize all the thinking you do before you start to solve the problem, e.g. what you are going to do, when, why, and how. Even second-guessing yourself is important to verbalize aloud, e.g., "I think I should use that long, complicated formula we were using a couple of weeks ago. What was it called, the quadratic equation? No, maybe not. Maybe I'm supposed to use the formula we used in class yesterday."
4. Verbalize all thoughts during the problem-solving, e.g., "Okay, I'm almost through with this division problem. Now that I have the answer, all I have to do is multiply to check and see if my answer is right."

As well as problem solver's responsibility listeners shares certain tasks during the instruction. According to Hartman these tasks include:

1. Think along with the problem-solver. Follow every step and make sure you understand every step. If not, ask a question. Have the problem-solver identify and define important terms, variables, rules, and procedures. Make sure the problem-solver vocalizes all the steps and does all the work. If the problem-solver skips over a step without thinking aloud, ask him or her to explain the missing thought.
2. Do **NOT** work the problem out independently. Listen to and work along with the problem solver.
3. Never let the problem solver get ahead of you. When necessary, ask the problem-solver to wait so you can check a procedure or computation and catch up. If the problem-solver is working too fast, slow them down so you can follow carefully, analytically, and accurately.
4. Check the problem-solver at every step. Don't wait for the answer. Check everything - each computation, diagram, and procedure. In the back of your mind, constantly ask yourself, "Is that right? Did I check that?" To promote precise thinking, have the thinker carefully define important terms and variables.
5. If you find an error, avoid correcting it. Point it out and try to get the problem-solver to correct it. If he or she gets stuck, ask questions to guide thinking in the right direction. If necessary, give some suggestions, hints or partial answers. Give the answer only as a last resort.

The second teaching approach is pair problem solving a higher level thinking (metacognitive), self monitoring strategy, which uses think aloud process. Problem solver and listener can be student or teacher. It is generally used to provide students feedback on their problem solving and thinking process by helping them to think about their own problem solving. (Hartman). Group problem solving as an instructional strategy has some barriers in student learning such as time limitation, which prevents teachers to analyze student's problem solving behaviors. Pair or small group instruction can be applies as an alternative method. Asking students to write down their cognitive process and problem solving strategies both provides accountability and skill development (Lavoie, 1995).

Taconis, Ferguson-Hessler and Broekkamp (2001) points out that traditional teaching approaches focuses on the sequence of the problem solving steps rather than cognitive strategies necessary to process through problem solving. Research suggests that age is a factor on children's reasoning ability and skills as well as inquiry being effective on analytical skills. A problem includes several dimensions; complexity, familiarity, closed- or open-type, amount of information provided, and type of cognitive activities required for the solution. Problem solver needs to be involved in knowledge and skill based cognitive activities to reach an outcome and an internal product which is learning. A study conducted by Taconis, Ferguson-Hessler and Broekkamp (2001) investigating a variety of teaching strategies suggests that studying worked examples stimulating knowledge and thinking skills and concept mapping may contribute to mastery of science problem solving and cognitive activities increasing the knowledge base has great importance.

Resources

Following are selected resources related to mathematical problem solving. Resources include online Web resources, and resources indexed by the ERIC system.

Web Resources:

Similar resources can be found by using the Google search engine and search terms such as science OR scientific "problem solving."

The NASA Why? files

http://whyfiles.larc.nasa.gov/treehouse.html

This site, part of the NASA WHY? FILES series, was developed by NASA for grades 3-5 and features online problem-based investigations and at-home activities complementary to the NASA Why? Files television broadcast series.

21st Century Problem Solving

http://www.hawaii.edu/suremath/home.html

This site is directed to students, teachers, administrators, and other professionals interested in problem solving. This site is dedicated to promoting problem solving literacy and covers problems related to algebra, physics, and chemistry.

PCPOW (Physics and calculus problems of the week)

http://www.kent.wednet.edu/pcpow/

This site is designed and maintained by two AP Calculus and AP Physics teachers as a way to foster an increased enjoyment of advanced math and science concepts through problem solving with a competitive twist.

Inquiry & Problem Solving

http://www.enc.org/topics/inquiry/

Articles on this theme show how classroom teachers encourage their students to become inquirers and problem solvers. Related materials from the ENC collection are also available.

ERIC Citations:

Barrow, L. H., et al. (1996). Evening science: Solving science problems. *Science and Children,* 34 (2), 20-23,49.

Describes the Evening Science program, a science enrichment program for K-6 students and their parents where students and parents engage in problem-solving activities that encourage science learning for the whole family. EJ 531 544.

Candela, A. (1997). Demonstrations and problem-solving exercises in school science: Their transformation within the Mexican elementary school classroom. *Science Education,* 81 (5), 497-513.

Argues that scientific knowledge in school is a social construct in which curriculum proposals are simply points of departure that are transformed by the social interaction that takes place within the classroom. Considers discourse as playing an important role in knowledge construction. EJ 552 951.

Gallet, C. Problem-solving teaching in the chemistry laboratory: Leaving the cooks... *Journal of Chemical Education,* 75 (1), 72-77.

Gives a step-by-step description of a problem-solving microscale chemistry laboratory to show how this implementation of problem-solving teaching can be put into practice. Pedagogical and practical issues are discussed. EJ 558 862.

Gatt, S. (2000). Problem-solving in primary science. *Primary Science Review,* 61, 8-10.

Discusses the use of problem-solving activities in elementary-level science lessons. Sample activities include "Sparkling Christmas Presents," "Who Left the Message?," and "Parachute Descent". EJ 602 402.

Klinger, A. (1998). The Earth is flat, and I can prove it! *Science Scope,* 21 (4), 35-36.

Describes an educational program that asks students to attempt to prove that the earth is spherical and that it rotates. Presents tips to pique student interest and charts related to sensing the spin, nonrotation notions, flat earth fallacies, evidence that the earth is spherical and rotates, and the role of watersheds in proving that the earth rotates. EJ 557 534.

Laing, M. (1996). Bring back equivalent weight--If you want the kids to "think"! *Journal of Chemical Education,* 73 (11), 1007-1012.

States that science courses present modern derived knowledge to be learned and applied to exercises. Proposes looking at the process of problem solving as an exercise in the scientific method against a historical background in social context with information about personalities involved. Uses an example moving from vague concept of element to a correct understanding of atomic weight and valence. EJ 536 546.

Moreno, N. P., Griffin, R. A., Denk, J. P., & Jones, W. (2001). Real-world science: Achieving better returns in student learning. *School Business Affairs,* 67 (8), 17-20.

Baylor College of Medicine, a Houston-area school district, and the Harris County Department of Education implemented a real-world science-instruction model in all district K-4 classrooms. The 7-year Environment as a Context for Learning Science (ECOS) program provokes student interest by centering on locally relevant environmental issues. EJ 634 666.

Taconis, R., Ferguson-Hessler, M. G. M., & Broekkamp, H. (2001). Teaching science problem solving: An overview of experimental work. *Journal of Research in Science Teaching,* 38 (4), 442-468.

Performs analysis on a number of articles published between 1985 and 1995 describing experimental research into the effectiveness of a wide variety of teaching strategies for science problem solving. Identifies 22 articles describing 40 experiments that met standards for meta-analysis. Indicates that few of the independent variables were found to characterize effective strategies for teaching science problem solving. EJ 625 455.

Information Problem Solving

One of the side effects of the information age is that it is becoming increasingly difficult to find a particular piece of information amongst all the information available. While electronic networks, databases, and the World Wide Web have brought a world of information to our fingertips, the information is not organized in any consistent way from place to place. So, one of the emerging new skills of literate citizens is how to search through all the information available to fine what they need. So, an important new area of problem solving is information problem solving. That is, the simple act of gaining access to information on one's own required use of information technologies, information seeking skills, and the ability to evaluate the quality of the information.

Though it is relatively easy to use electronic search engines to find an abundance of information, a lack of skill in information problem solving will likely lead students and others to retrieve less meaningful information. So, students must learn both how to select the right search engine for a particular problem and how to use the various features of search engines to find relevant information. Though information problem solving will become increasingly important for everyone, people in the library sciences and information technologies have provided the early development of information problem solving models and resources up to this point. Following are resources and guidelines developed for use by these professionals.

Resources

Following are selected resources related to mathematical problem solving. Resources include online Web resources and resources indexed by the ERIC system.

Web Resources:

Similar resources can be found by using the Google search engine and search terms such as "information problem solving."

Teaching Information Literacy: The Big Six Skills Approach to Information Problem Solving

http://www.itrc.ucf.edu/webcamp/final_projects/barney/big6.html

The information used to develop this website is taken from "Computer Skills for Information Problem-Solving: Learning and Teaching Technology in Context, ERIC Digest" (1996, March), prepared by Michael B. Eisenberg and Doug Johnson for the ERIC Clearinghouse on Information & Technology, Syracuse, NY.

The Big 6 Skills Information Problem Solving Approach

http://www.geminfo.org/Consortium/Members/Big6.html

This official Big6 Web site provides overview and in-depth information about the Big6. The site is aimed at educators, administrators, parents, and anyone else interested in helping students to learn and use essential information and technology skills.

Information Problem Solving Models K-12

http://www.bcps.org/offices/lis/models/

The Information Problem Solving Research Models (IPSRM), developed by Baltimore County Public Schools, represent an exciting new way to guide student research toward higher-level thinking that fully utilizes electronic resources. The research models were developed by teams of library media specialists and teachers during the 1998-2001 Summer Curriculum Workshops in the Baltimore County Public Schools. Students who use the self-guided online research lessons are challenged to employ thoughtful reading, analysis, evaluation, and synthesis of information to create answers, not just find them.

The IPSRMs are designed as web pages that present students with a clear research structure, including a research scenario, a learning task, rubrics and scoring tools, directions for use of various media resources, links to useful web sites, production of a product, and reflection. Their availability on the Internet also serves to make curriculum information accessible to parents and the general public.î

Big Six, Super Three & Information Problem Solving

http://www.nhps.net/worthingtonhooker/bigsix.htm

Teacher information related to these approaches to research.

Teaching Information Problem Solving in Primary Schools: An Information Literacy Survey

http://www.ifla.org/IV/ifla63/63moop.htm

Advances in information technology have resulted in pressure to examine the way educators approach the task of developing children's information literacy. The following reports on work in progress to identify challenges faced in integrating information skills across the primary school curriculum. The stated information literacy intentions of schools, gathered from policy documents and through semi-structured interviews with key people, are compared with teachers' responses to a questionnaire. The questionnaire focused on their understanding of information skills, resource-based learning, the role of the library and the teacher with library responsibility.

Differences emerged in the clarity and coherence of the vision held for information literacy within the participating schools and in staff understanding of the nature of information skills, the role of the library and resource-based learning. Teachers' expectations for children's ability to find and use information are also explored.

Information Problem-Solving Projects

http://wwwgen.bham.wednet.edu/onlinepr.htm

Online research projects.

Information Problem Solving: "The Big Six"

http://www.jlhs.nhusd.k12.ca.us/Classes/Science/Research.html

This page describes the six main research skills necessary to becoming a successful researcher. Each skill is broken down into several detailed steps that you can follow while researching a particular topic in science, or any other subject.

ERIC Citations:

Berkowitz, R. (1998). Helping with homework: A parent's guide to information problem-solving. *Emergency Librarian,* 25 (4), 45-46. Worthington, OH: Linworth Publishing.

This book about using the Big6 information problem solving process model in elementary schools is organized into two parts. Providing an overview of the Big6 approach, Part 1 includes the following chapters: "Introduction: The Need," including the information problem, the Big6 and other process models, and teaching/learning the Big6; "The Big6 Process and Skills," including the Big6 levels (i.e., the conceptual level, the Big6 Skills, and the Little12 sub-skills) and the Super3 (a simplified version of the Big6 for younger students); "Technology with a Big6 Face," including technology and the Big6, examples of technology in Big6 contexts, and the Big6 and the Internet; "Implementing the Big6: Context, Context, Context," including process and curriculum contexts, analyzing curriculum from a Big6 perspective, and planning and plans; and "Assessment of Information & Technology Skills," including effectiveness and efficiency, forms and context for assessment, ways of assessing, Big6 scoring guides, and self-assessment. Part 2 presents the Big6 in action through TIPS (Teaching Information Problem-Solving) explanations, sample integrated lesson plans, and Big6 activities for each of the Big6 stages: (1) Task Definition; (2) Information Seeking Strategies; (3) Location & Access; (4) Use of Information; (5) Synthesis; and (6) Evaluation. TIPS for introducing the Super3 are also provided. Appendices include a comparison of information skills process models and the "Big6 Song." EJ 565 458.

Berkowitz, R. E., et al. (1988). When information skills meet science curriculum: A cooperative effort. *School Library Media Activities Monthly,* 4 (10), 28-33.

Presents an instructional unit designed to integrate information problem-solving skills within the context of honors biology and chemistry curriculum content. Audience, rationale, general objectives, skills objectives, science content objectives, unit organization, planning time, activities, evaluation, and materials are outlined. EJ 372 422.

Eisenberg, M. B., & Berkowitz, R. E. (1996). *Helping with homework: A parent's guide to information problem-solving.* Syracuse, NY: Information Resources Publications, Syracuse University, Center for Science and Technology.

The purpose of this book is to help parents become partners in their childrens' success in school by offering them practical ways to help with homework and assignments. Parents can use the Big Six Skills information problem-solving process to effectively deal with the abundance of information available from many sources and guide their children through school assignments. The Big Six Skills apply to any problem or activity that requires a solution or result based on information: task definition, information seeking strategies, location and access, use of information, synthesis, and evaluation. There are six chapters in the book: (1) "The Big Six Approach: A Framework for Helping Children"; (2) "What Your Children Face Every Day in School: Assignments"; (3) "The Big Six Applied: A Framework for Helping Children with Homework"; (4) "Computers, the Internet, and Other Technologies: Can They Really Make a Difference?" (5) "Assisting with Assignments: Examples from Various Subjects"; and (6) "Bringing It All Together: A Parent Conversation with Mike and Bob." Appendices include the Big Six Assignment Organizer, applying the Big Six to sample homework assignments, Big Six Skills overhead transparency masters and bookmark, background information on the Educational Resources Information Center (ERIC), and a selected bibliography of ERIC documents. ED 418 699.

Kasowitz, A. S. (2000). *Using the Big6™ to teach and learn with the internet.* Worthington, OH: Linworth Publishing.

This book is designed to prepare information mentors-educators, parents, educational concept developers, subject-matter experts, and others who guide K-12 students to information literacy to provide instruction, guidance, and services to teach K-12 students how to solve information problems using a variety of information tools and resources. The introduction describes the challenges involved in learning and teaching in a technology- and information-rich society, and presents the Big6 information problem-solving process as a vehicle for information mentors to help students develop information literacy skills. Chapter 1, "Planning Instruction Using the Internet," provides a background on using the Internet in instruction, and guidelines for planning instructional opportunities that incorporate the use of the Internet within an information problem-solving approach. Sample less plans and projects are given. Chapter 2, "Coaching K-12 Students with the Internet," presents issues involved when using the Internet with K-12 students and offers suggestions for successful information mentoring experiences. Chapter 3, "Communicating with Students on the Internet," provides background on the role of telecommunications activities in education, particularly Ask-an-Expert services and telementoring projects, and offers suggestions for promoting the use of information problem-solving through online communications. Chapter 4, "Designing and Providing Content on the Internet for K-12 Students," describes some different types of Internet resources designed for K-12 students and offers guidelines and examples for designing content that enhances learning and promotes an information problem-solving approach. Each chapter includes a summary, annotated list of resources, and a worksheet for applying the principles of the chapter. Appendices provide information on: getting started on the Internet (courses and resources); searching on the Internet; evaluation of Internet sites; academic standards on the Internet; locating and designing school Web pages; citing Internet sources; online safety; and designing Web sites. ED 449 781.

Sine, L., & Murphy, B. (1992). Teaming to teach the information problem-solving process. *School Library Media Activities Monthly,* 9 (3), 30-31,35.

Explains a problem-solving format developed by a school media specialist and first grade teacher that used the framework of Eisenberg and Berkowitz's "Big Six Skills" for library media programs. The application of the format to a science unit on the senses is described. EJ 602 402.

Creative Problem Solving

The model for solving problems in creative way has been introduced by Osborn in 1952. Since then, it has become one if the most popular approaches used in stimulating critical thinking (Hillis & Puccio, 1999). Isaksen and Dorval describes three major phases of the development of a Creative Problem Solving Process (CPS) framework throughout the history of CPS. The first phase was developed by Alex Osborn during the early years of CPS. The purpose was to model creative process to have a better understanding of creative talent effectively. Five CPS stages were outlined based on the research; fact-finding, problem-finding, idea-finding, solution-finding, and acceptance-finding. The conclusion of this phase was that people have different learning styles and ways of using the CPS model. Simplistic description and graphing representations of CPS were the outcome of the first phase. The second phase focused on understanding individual differences for a universal application of the model by exploring the social roles. The addition of mess-finding as the sixth stage, development of a balance between creative and critical thinking phases, and renaming fact-finding as data-finding for the implementation of impression, feeling and opinions occurs during the second phase.

Three main components of CPS were organized for a more accurate understanding of the problem solving process; understanding the problem, idea generation and planning for action. Third phase presents a new approach to the view of CPS by breaking the three components apart for a more descriptive approach. This phase allows effective inquiry. (Isaksen & Dorval)

Resources

Following are selected resources related to mathematical problem solving. Resources include online Web resources and resources indexed by the ERIC system.

Web Resources:

Similar resources can be found by using the Google search engine and search terms such as "creative problem solving."

Critical Thinking & Problem Solving Skills

http://falcon.jmu.edu/~ramseyil/critical.htm

This page has general information, lesson plans and bibliographies to help educators interested in higher order thinking skills.

Project Renaissance, Creative Problem-Solving Techniques

http://www.winwenger.com/mind.htm

Creative Problem Solving: Use Data Analysis and Collaborative Approaches

http://www.changedynamics.com/probsolv.htm

Although there are numerous tools and techniques for analyzing problems, there are two major approaches that greatly simplify the problem-solving process.

Odyssey of the Mind

http://www.odysseyofthemind.com/

The Odyssey of the Mind School Program fosters creative thinking and problem-solving skills among participating students from kindergarten through college. It features an annual competition component at local through international levels. Students solve problems in a variety of areas, from building mechanical devices such as spring-driven vehicles to giving their own interpretation of literary classics. Through solving problems, students learn lifelong skills such as working with others as a team, evaluating ideas, making decisions, and creating solutions while also developing self-confidence from their experiences.

Creative Problem Solving ~ Thinking and Learning

http://www.mdk12.org/practices/good_instruction/projectbetter/thinkingskills/ts-15-16.html

Rationale.

Creative Problem Solving in Math

http://www.uh.edu/hti/curriculum_units/2000_Seminars/vol2_creativity/HTML/Pepkin_Unit.htm

This document can also be downloaded as a PDF file at

http://www.uh.edu/hti/curriculum_units/2000_Seminars/vol2_creativity/PDF/Pepkin_Unit.pdf.

Creative Problem Solving in Science and Technology

http://www.soe.ku.edu/sites/science_ed/CPS1.htm

Center for Creative Learning, Inc.

http://www.creativelearning.com/bibliography.htm

Creative Problem Solving Bibliography

http://www.stolaf.edu/other/snap/creabib.html

Creative Problem Solving Activities

http://www.criticalthinking.com/www/CreativeProblemSolving/cps.html

Planning for Problem Solving Instruction

http://www.ncrel.org/sdrs/areas/issues/content/cntareas/math/ma6plan.htm

The Critical Thinking Community

http://www.criticalthinking.org/

Resources for education professionals interested in increasing critical thinking skills.

ERIC Citations:

Alamaki, A. (1998). *Technology education in elementary school: Why and how?* Paper presented at the annual meeting of the International Technology Education Association (60th, Fort Worth, TX, March 8-10, 1998).

This paper discusses using technology education in the elementary school to encourage innovation and adaptation, technological literacy, and creative problem solving. Technology education is seen based on hands-on activities where pupils make things and become familiar with their technological environment. The hands-on activity should evolve like the creative problem-solving process, which begins with an idea and ends with a product or solution via searching, trying, and realizing. If hands-on science is reviewed in a larger context than merely arts or aesthetics, one can learn to understand how man creates his technological world. If technology education is viewed from the viewpoint of students, one notices that students must be taught the knowledge, skills, readiness, and values which they will need later in life. Postmodern society and economic life will require young adults to have the capacity for innovation and adaptation, technological literacy, and creative problem-solving skills. Includes an example of teaching about different transportation systems in order to plan production of water vehicles. ED 418 880.

Allen, D. (1995). Creative problem solving. *Teaching PreK-8,* 25 (4), 18-25.

Reviews several student-tested computer software programs for math instruction: Gameco's "Word Problem Square-Off"; "MathKeys" by MECC and Houghton-Mifflin; Davidson and Associates' "The Cruncher"; and Sunburst Communications' "Exploring Mathematics with Technology: Number and Operation Sense." EJ 495 346.

Black, D. (1994). Resolving mysteries: A guide to creative problem solving. *Instructional strategies series, No. 17.* Regina, Saskatchewan, Canada: Saskatchewan Book Bureau.

This booklet describes a process for creative classroom problem solving that can be adapted to many types of problems. The booklet's five sections correspond to five phases of the problem solving process. Each section begins with a description of the phase and then provides suggestions for using that phase in the classroom. The sections conclude with case studies that illustrate the way a basic problem-solving process can be modified to suit a number of different types of situations. "Your Turn" boxes throughout the booklet will allow the reader to resolve a problem in his or her own life. The five sections address: (1) recognizing that an unresolved situation exists as a first stage of problem solving; (2) identifying the problem; (3) finding solutions including tools for generating ideas, suggestions for when the process gets stuck, and problem solving with students; (4) choosing from among alternatives including identifying criteria, decision making, adapting criteria, and problem solving with students; and (5) taking action such as looking at the results of the action and working with students in this phase. ED 380 426.

Bohan, H., & Bohan, S. (1993). Extending the regular curriculum through creative problem solving. *Arithmetic Teacher,* 41 (2), 83-87.

Uses ancient Egyptian numeration system in a new setting to extend the concepts of base, place value, and correspondence. Discusses similarities and differences between the Egyptian and decimal systems. Students are asked to propose changes to make the Egyptian system easier. EJ 474 911.

Eberle, B., & Stanish, B. (1996). *CPS for kids: A resource book for teaching creative problem-solving to children.* Waco, TX: Prufrock Press,

This book, designed for grades 2-8, teaches students the creative problem-solving (CPS) method. Each step in the process is outlined in detail and is illustrated with 30 reproducible classroom activities. The levels of the problem-solving approach addressed in the book include: (1) "Sensing Problems and Challenges"; (2) "Fact Finding"; (3) "Problem Finding"; (4) "Idea Finding"; (5) "Solution Finding"; and (6) "Acceptance Finding." A description and overview of each level precede student activity sheets and acquaint the teacher with the particular processes involved. ED 406 291.

Edmund, N. W. (2000). *The scientific method today: Your guide to the complete method of creative problem solving and decision making.* SM-14. Ft. Lauderdale, FL: Norman W. Edmund.

This booklet introduces the SM-14 (Scientific Method) Formula and lists characteristics of the scientific method. A history and progression of the scientific method is also featured. Contents include: (1) The Basic Principles of the Scientific Method; (2) Submitting Ideas Incorporating SM-14; (3) The 11 Stages of the SM-14 Formula and Famous Examples; (4) The Three Supporting Ingredients of the SM-14 Formula; (5) Everyday Problems and Decisions and Explanations of Methods and Method; (6) Helpful Information on Creativity, Decision Making, Invention, and Scientific Management; and (7) Guide and Worksheet for Applying SM-14. ED 445 921.

Harkow, R. M. (1996). *Increasing creative thinking skills in second and third grade gifted students using imagery, computers, and creative problem solving.* Master's final report, Nova Southeastern University.

This practicum project used a combination of strategies to improve creative thinking skills in second- and third-grade gifted students. Sixteen students were targeted for the intervention. Over a 12-week implementation period, students participated in 90-minute interventions twice weekly. The intervention was comprised of 30-minute creative problem-solving encounters with peers; 30 minutes of computer software use to produce original writing, and to experiment and create in open-ended settings; and a 30-minute period of activities alternating between relaxation and imagery exercises and the use of imagery in creative writing. There were four objectives to the intervention: (1) increasing verbal and figural creativity; (2) increasing figural and verbal fluency; (3) increasing figural and verbal originality; and (4) increasing verbal flexibility. The Torrance Tests of Creative Thinking, Figural and Verbal Models and the Inventory of Creative Behaviors were used to assess the impact of the intervention. Informal teacher observations were conducted throughout implementation. The Inventory of Creative Behaviors was completed weekly and at the conclusion of the implementation period. Findings indicated that the proposed number of students met the projected percentage of increase of 80 percent or above in overall figural and verbal creativity, verbal originality, and verbal flexibility. Fewer than the proposed number of students met the projected increase in figural and verbal fluency and figural originality, although all students showed significant increases in these areas. (Eighteen appendices include School Profile, Inventory of Creative Behaviors, software evaluations, schedule of computer use, and pre- and posttest results and comparisons. ED 405 982.

Huber, R, A., Smith, R. W., & Shotsberger, P. G. (2000). *The impact of a Standards guided equity and problem solving institute on participating science teachers and their students.*

This study examined the effect of a teacher enhancement project combining training on the National Science Education Standards, problem solving and equity education on middle school science teachers' attitudes and practices and, in turn, the attitudes of their students. Participating teachers reported changes in their instructional methods that included increases in the use of cooperative learning, scientific inquiry, creative problem solving and questions directed toward higher order cognitive processes. Participating teachers' students indicated positive changes in attitudes and views toward science, particularly in the responses of female students and students of color. ED 442 621.

Jackson, J. B., Crandell, L., & Menhennett, L. (1997). Future problem solving: Connecting the present to the future. In *China-U.S. conference on education. Collected papers.* (Beijing, People's Republic of China, July 9-13, 1997).

To survive in the 21st century, today's student must develop the thinking skills necessary to adapt to a transforming world and learn how to be a creative problem solver. Future Problem Solving (FPS) prepares the student of today for tomorrow through an educational program that enables students to apply a constructive, deliberate process for solving problems. The FPS approach, based on the Creative Problem Solving (CPS) framework, has three components: (1) understanding the problem; (2) generating ideas; and (3) planning for action. FPS provides competitions for which students identify a real-life need and use the FPS model to create a solution. As a curriculum program, FPS provides a framework for solving complex, real-life challenges. The foundation tools of the CPS process, creative thinking and critical thinking, must both be employed for the results to be productive. In July 1997, four Arizona FPS students and the Arizona FPS affiliate director presented FPS at the China-U.S. Conference on Education in Beijing. This paper describes the renewed partnership of FPS and CPS, student interpretations of FPS, and an overview of the Chinese and American students' problem-solving activity. ED 425 404.

Larson, K. (1993). Art and environment: An integrated study on the web of life. In *Art, science & visual literacy.* Selected readings from the annual conference of the International Visual Literacy Association (24th, Pittsburgh, PA, September 30-October 4, 1992).

Art education combined with the sciences can be a leading force in raising the consciousness of students and teachers to the needs of our fragile planet. By teaching environmental aspects to the visual arts curriculum, we will also be teaching that one of the similarities we all share is the need for beauty. The sciences have traditionally carried the burden of teaching about the environment and ecological problems, but what seems to have gone unrecognized is the creative brainstorming that often precedes the scientific process. A recent example of creative problem-solving that combines art and science is the design of a tree-top raft created by a French architect. The geometrically designed raft balances on top of the trees in the French Guiana rainforest, thus allowing scientists access to "walk" across the canopy to study plants, insects, and animals in a previously unexplorable area. Art and science can also be merged in a curriculum that includes these approaches: (1) looking at the problem; (2) creative drama and imagining; (3) brainstorming and thinking time; and (4) creating. Three lesson plans are given as suggestions for environmentally aware art and science classes. A list of 13 resources for teaching environmental education is included. ED 363 335.

Loewen, A. C. (1995). Creative problem solving. *Teaching Children Mathematics,* 2, 96-99.

Discusses and illustrates the difference between traditional and creative problem solving. EJ 514 079

Mikovec, A. E., Dake, & Dennis M. (1995). Tying theory to practice: Cognitive aspects of computer interaction in the design process. In *Eyes on the future: Converging images, ideas, and instruction.* Selected readings from the annual conference of the International Visual Literacy Association (27th, Chicago, IL, October 18-22, 1995).

The new medium of computer-aided design requires changes to the creative problem-solving methodologies typically employed in the development of new visual designs. Most theoretical models of creative problem-solving suggest a linear progression from preparation and incubation to some type of evaluative study of the "inspiration." These models give a communicable structure to infinitely variable creative experiences, but that perspective may need to be altered in the integration of computer applications into design education. In its infancy, computer-aided design merely saved engineering students the tedium of computation. Later on, computers were used to assist in drafting. Currently the computer can help with many aspects of visual design, including allowing for three-dimensional study models and providing access to helpful newsgroups and remote resources through the Internet. As long as students have the advantages of some previous knowledge of the programs and of appropriate hardware, computer applications can help them represent their ideas graphically. The computer-aided design environment is characterized by several qualities that require a move away from the linear problem-solving paradigm: (1) interactivity with programs, or even with the Internet, provides a cycle of immediate feedback which does not lend itself to assembly-line design or learning; (2) the visually mediated form of thinking is more holistic than linear; (3) the open-ended, discovery-oriented dynamic seems to operate without strict rules of causation; and (4) its ability to empower individual designers to make decisions conflicts with older views of the designer as a detached observer. The new model for creative problem solving is a feedback loop, an ongoing cyclical process of discovery and evaluation. Three separate studies of computer-aided design studios from fall 1993 to summer 1995 found these qualities at work with varying degrees of success; in some cases, interactivity was slow to develop, while in others, lack of personal empowerment or too much concentration on product over process was a problem. In some situations, moreover, the technology was not being used to its full potential. Future research is recommended. ED 391 491.

Nichols, T. M. (1993). Effects of problem-solving strategies on different ability levels. Paper presented at the annual meeting of the Mid-South Educational Research Association (22nd, New Orleans, LA, November 10-12, 1993).

To determine if differing ability levels will affect the acquisition of problem-solving skills and self-esteem as a result of participation in two approaches to teaching problem-solving skills, a study was conducted with sixth graders in a posttest-only control group experimental design. Subjects were 102 sixth graders randomly assigned to 5 classes. Two classes participated in the Creative Problem Solving (CPS) for Kids approach to teaching problem solving. Two classes received computer-assisted instruction in problem-solving designed by the Minnesota Educational Computing Consortium, and one class was a control group. Both approaches consisted of five 3-minute lessons per week for 6 weeks. Results suggest that thinking-skills instruction does impact the development of creative and critical thinking and that the acquisition of these skills has a positive effect on self-esteem. The study also provides evidence that the length of training is an important consideration in providing thinking-skills instruction, and that such instruction should be an integral part of the curriculum rather than a supplementary or isolated program. In addition, thinking-skills instruction is appropriate for students at all ability levels. Seven figures and 12 tables present study findings. ED 366 632.

Piirto, J. (1998). *Understanding those who create.* Second Edition. Scottsdale, AZ: Gifted Psychology Press.

This book synthesizes research findings on creativity and talent development. Part 1, "Definitions and Processes of Creativity," discusses the definition of creativity, creativity and psychology, federal definitions of giftedness and creativity, psychological research on creativity, traditional theories of the creative, common descriptions of the creative process, newer theories of the creative process, creativity as the process of a life, the creative process as cognitive science, and the creative problem-solving process. Part 2, "Creativity Assessment and Training," addresses creativity testing, validity and reliability of creativity tests, studies of significant results, using personality questionnaires, personality and behavior checklists, promising testing practices, and creativity training. Part 3, "Personality and Intellectual Characteristics of Creative People in Various Domains," discusses predictive behaviors and crystallizing experiences of visual artists and architects; creative writers and children with extraordinary writing talent; creative scientists, mathematicians, inventors and entrepreneurs; musicians, conductors, and composers; and physical performers, including actors, dancers, and athletes. Part 4, "How To Enhance Creativity," provides strategies for encouraging creativity through motivation and schooling and ways that parents and teachers can enhance creativity in children. Each of the 12 chapters concludes with a list of summary statements that summarize insights contained in that chapter. ED 420 961

Polland, M. J. (1996). Mental imagery in creative problem solving. Ph.D. Dissertation, Claremont Graduate School.

In order to investigate the relationship between mental imagery and creative problem solving, a study of 44 separate accounts reporting mental imagery experiences associated with creative discoveries were examined. The data included 29 different scientists, among them Albert Einstein and Stephen Hawking, and 9 artists, musicians, and writers, including Leonardo da Vinci and Richard Wagner. Thirty-three of the incidents were reported by the subjects themselves, and 11 were written by biographers and historians. The study analyzed reports of mental imagery according to the following three factors: (1) possible causes for the reported perception of mental imagery; (2) what perceptual modalities were involved in the reported mental imagery; and (3) at what stage in the problem solving process imagery was reportedly involved. Findings included the following: (1) mental images occurred more often in a spontaneous way when the subjects were occupied with routine behaviors; (2) imagery that occurred in the visual modality was the single most reported factor, which may reflect the fact that the research focused on scientific fields that tend to rely on visual representations; and (3) mental imagery was reported to occur most frequently during the later stages in the creative process. ED 393 593.

Proctor, R. M. J. (2001). Enhancing elementary students' creative problem solving through project-based education. In *Building on the future. NECC 2001: national educational computing conference proceedings* (22nd, Chicago, IL, June 25-27, 2001)

For full text:
http://confreg.uoregon.edu/necc2001/program/.

This paper reports on one dimension of a longitudinal study that researched the impact on student creativity of a unique intervention program for elementary (year 6 and year 7) students. The intervention was based on the Australian National Profile and Statement for the curriculum area of Technology. The intervention program comprised project-based, collaborative, and thematically-integrated curriculum units of work that incorporated all eight Australian Key Learning Areas (KLAs). A pre-test/post-test control group design investigation was undertaken with 520 students from seven schools and 24 class groups that were randomly divided into three treatment groups. One group (10 classes) formed the control group. Another seven classes received the year-long intervention program, while the remaining seven classes received the intervention, but with the added seamless integration of information and communication technologies (ICTs). The effect of the intervention on the personal dimension of student creativity was assessed using the Creativity Checklist, an instrument that was developed during the study. The results suggest that the purposeful integration of computer technology with the intervention program positively affects the personal creativity characteristics of students. ED 462 941.

Schramm, S. (1997). *Related webs of meaning between the disciplines: Perceptions of secondary students who experienced an integrated curriculum.* Paper presented at the annual meeting of the American Educational Research Association (Chicago, IL, March, 1997).

The purpose of this inquiry was to determine the perceptions of certain secondary students in the Greater Cincinnati (Ohio) area who experienced an integrated curriculum that combined the subjects of geometry and visual art. A mathematics teacher and a visual arts teacher collaborated to facilitate student discovery by implementing creative problem-solving strategies that make unique connections between the traditional subjects of geometry and art. Specifically, students explored paper engineering concepts while designing three-dimensional pop-up greeting cards. Understanding of how secondary students perceive integrated curriculum in the context of human experience was generated through surveys and semi-structured interviews with the students who participated in the integrated project. Since many students often see little relevance in school life, the study also addresses how integration connects subject areas in ways that reflect real world applications of mathematics and art in industry. Implications of the study are based on the cross-curricular connections students discover as they bridge the gap between their classrooms and the real world of problem solving. Appendices include survey and feedback forms. ED 407 258.

Stanish, B., & Eberle, B. (1997). *Be a problem-solver: A resource book for teaching creative problem-solving.* Waco, TX: Prufrock Press.

This book provides an overview of the creative problem-solving process and exercises to put the process the work. The illustrated, reproducible pages guide students through each step of the problem-solving process by using evaluation grids to track their ideas, solutions, and plans. The activities can be used in a variety of ways including for the whole class, in a learning center, as take home work, for display materials for generating student interest and motivation, or as warm-up activities for problem-solving competitions. ED 405 273.

A General Model of Problem Solving

The successful use of problem solving methodologies has become a very important process skill that most teachers diligently try to develop. This skill is identified in both the Principles and Standards for School Mathematics (1989) (PSSM) and the National Science Education Standards (NSES). The PSSM states that

"In grades 9-12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can—

- use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;
- apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems form situations within and outside mathematics;
- apply the process of mathematical modeling to real-world problem situations." (1989 NCTM Standards)

In this Standard, problem solving is seen as being a major part of mathematics education itself. It gives the students an opportunity to use and develop other math skills. It also stresses that the problems should be real-world problems situations so that the students can use their problem solving skills in situations that they will later be able to adapt to their own lives.

The National Science Education Standards note that learning science is an active process and therefore implies both physical and mental activities. The students should be using both hands-on and minds-on activities (National Science Education Standards, Chapter 2). Students are expected to engage directly in problem solving in order to help satisfy the active learning of science.

An important aspect of the problem solving process is the use of modeling. Swetz (1989) describes two major types of models: physical and theoretical. Physical models often bring to mind concrete, miniaturized examples of their subject matter such as miniature towns or prototype engineering projects. But physical models can also be full size as when a metal worker makes a sand mold for producing cast metal moldings. The worker uses a full size model which allows him to make changes to the product that will easily be seen. Physical models are typically made out of materials that are simple to work with and also that allow for changes to be made easily.

Theoretical models "are collections of principles or rules that accurately describe the behavior of a phenomenon in the mind of an observer" (Swetz, 1989). These models are based on concepts understood by the problem solver and can be limited by the observer's knowledge. Theoretical models are also useful when conditions such as cost, size, or time constraints do not allow for the construction of a physical model. Theoretical models are used when new "theoretical" ground is being broken and concrete physical models cannot fully describe the new concepts being developed.

Other important modeling processes used in problem solving include illustrative modeling, analytical modeling, and simulation modeling (Colella, Klopfer, & Resnick, 2001). Illustrative modeling is designed to provide visualization of the model. These models are useful because they provide images to focus the concepts that are being studied. For instance, after students are presented with information concerning atomic theory, most teachers ask students to show their knowledge by drawing or recognizing an illustrative model of the atom. Illustrative models are very popular with students because they are able to put the information into their own conceptual visualizations. Illustrative models are also useful to teachers as an evaluative check on whether the students are understanding the information being studied.

Analytical models produce solutions based on specific rules or equations and a given set of conditions. These models are used when dealing with concepts based on mathematical equations such as velocity, acceleration, weather forecasting, and stock market projections. As students become better versed in their mathematical and science concepts, they will encounter these types of models more frequently.

Simulation models have become especially important since the advent of personal computers in the last 20 years. Simulation models allow for predictions over time and computers are able to provide these predictions through the fast calculations of the algorithms necessary for the predictions. These predictions allow for small changes to be made in variables to determine how these changes will affect the outcome of the simulation. Simulations such as these are run by the National Weather Service in an effort to predict the best possible scenarios for future weather. As can be seen by the weather forecasters percentages of correct weather prediction, simulation models do not necessarily predict events correctly every time.

Many different modeling processes are available and no one modeling process can be used for all systems. The correct model needs to be chosen to best fit the problem at hand that needs to be solved. Likewise, many problem solving models have been proposed in the last 100 years.

But before discussing various problem solving models, question that begs to be answered is, what is problem solving? Krulik and Rudnick (1987) define problem solving as:

"It is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation."

A situation is presented to the individual which presently has no resolve and when the individual has successfully secured a resolution and checked it for accuracy, the problem is solved.

The mathematical definition of problem solving is fairly straight forward. However, science educators, rather than provide a specific definition for problem solving, describe the process skills used during the problem solving situation and then classify it by the problem solving technique used in the situation (Helgeson, 1992). This method has led to a broad operational definition of problem solving based on science process skills and leaves the science education community with a definition of problem solving that few science educators can specifically define.

Even with these two different outlooks on problem solving, it is well agreed that problem solving is an extremely important process that needs to be taught to and learned by all students as they progress through their mathematics and science courses. But what steps are necessary to provide for resolving problems that encountered? Are the steps in problem solving always the same and can every problem be solved with the same set of steps?

Early models of problem solving used two distinct approaches: the traditional scientific method and an introspective creative method (Stephenson, 2000). John Dewey in 1910, described the scientific process as:

- Define the problem
- Suggest possible solutions and identify alternative solutions
- Critically think and reason about the solutions and use this information to solve the problem
- Test the solution and prove its correctness (Dewey, 1910)

This method had and has been used by scientists for years.

Wallas (1926) provided a creative process of problem solving and included the steps:

- Problem formulation and information gathering
- Incubation period—allowing the unconsciousness to explore the problem
- Illumination—working on solving the probe to gain understanding
- Verification—testing the solution for accuracy and correctness.

Later, Polya (1945) provided a problem solving model based on classroom experience. The steps included:

- Understanding the problem
- Devising a plan
- Carrying out the plan
- Looking back

Although not as seemingly comprehensive as the previous two models, it provides a broad view of the problem solving experience.

Bedell and Lennox (1996) provide a seven step problem solving process. These steps include:

- Problem recognition
- Problem definition
- Generation of alternative solutions
- Evaluation of alternative solutions
- Making a decision
- Implementation of the solution
- Verification of the solution's effect

Krulik and Rudnick (1995) provide a five step problem solving process. The steps include:

- Reading and thinking
- Exploring and planning
- Selecting a strategy
- Finding an answer
- Reflecting and extending

It is interesting to note that all methods include the recognition of the problem as the initial step and the reflection and verification of the solution as the final step. Some processes call for the production of possible alternative solutions while other models leave this step out. Although this step is left out in some models, clearly brainstorming is a function of each model and this would generate other possible solutions to the problem

The first step in any problem solving situation is recognizing what the problem is. Teamworks (Problem Solving, 1996), organizes the problem identification process into a four step process. These steps involve:

- Identifying possible concerns
- Determining mutuality of concerns
- Identifying complementary goals
- Identifying superordinate goals

Problem solving models can be as varied as the teaching methods used to present them. The wide range of models exist because each teaching method and situational problem would better lend itself better to a problem solving model best suited to that teaching method. Several problem solving models will be presented below as examples of the various methods available to the range of problems to be solved. These methods include Reflective Problem Solving, Dialectical Inquiry, Devil's Advocacy, Creative Problem Solving, Action Research Problem Solving, Analytic Problem Solving, and several problem solving teaching strategies.

The Reflective Problem Solving model emphasizes these important tasks: defining concepts, identifying needs, and recognizing and evaluating solutions (Problem Solving, 1996).

When using Reflective Problem Solving these steps are usually covered in a standard order.

- Define the problem.
- Analyze the problem.
- Establish criteria for evaluating solutions.
- Propose solutions.
- Take action.

This method centers around a straight forward, no-nonsense approach where individuals try to reflect on the problem with relation to the solutions at hand. The process begins with trying to identify symptoms of the problem, not the causes of the problem. The next step is to evaluate the data collected. The next goal is to make a list of MUSTS and WANTS. The MUSTS list consists of requirements the result must have, while the WANTS list are results that might be desirable but not necessary. The fourth step is to brainstorm followed by the writing of an action plan. This method can be used either by large groups or small individuals but is effectively kept to a step by step process.

Dialectical Inquiry is a conflict-based problem solving model where two separate and opposing recommendations are suggested and then these recommendations are debated (Problem Solving, 1996). The concept behind this model is to better understand the problem and examine each possible solution.

The method follows these steps:

- Divide the group into two advocate groups.
- The group develops a set of recommendations for solving the problem and includes reasons for the recommendations. This information is also give to the other group.
- After seeing the other group's recommendations, a set of counter-recommendations are produced.
- After both groups have organized their key assumptions, structured debate is ready to begin.
- The problem to be solved is stated before the group.
- A presenter is chosen to provide orally and in writing the facts, data and assumptions of the other group.
- After each side has presented, the groups debate both plans in an effort to find hidden or faulty agendas.
- After the debate is finished, the group agrees on which recommendations from the debate are the most plausible and works to fine-tune a result based on those plans.

Devil's Advocacy is another conflict-based model that relies on structured conflict. Well-documented arguments are subjected to intense evaluation by another group or person. It is assumed that only the strongest and best recommendations will survive the evaluation process. The steps include:

- Divide into two groups, one of which will be the devil's advocate.
- The first group develops recommendations to solve the problem and lists all assumptions and facts.

- These recommendations are then submitted to the devil's advocate group.
- The devil's advocate group evaluates each recommendation trying to find errors or problems with each effort.
- The first group goes back and evaluates each recommendation in light of the comments made by the devil's advocate group.

This process can continue until an adequate solution is found for the problem.

A Creative Problem Solving model includes the following steps towards providing a solution to the problem. These steps include:

- Fact finding.
- Problem finding. This includes identifying a problem which when solved, would provide a solution to major issues involved with the problem.
- Idea finding. Use brainstorming techniques to determine as many solutions to the problem as possible.
- Solution finding. Criteria are chosen and applied or evaluating the ideas.
- Acceptance finding. A plan is developed for implementing the solution while keeping in mind the appropriateness of the solution for the audience involved.

This method stresses positive attitudes and relies on self-evaluation of ideas to see if they clearly solve the problem. This method also stresses the importance of not only truly understanding the problem but also the underpinnings of the problem.

The Action Research Problem Solving model is a process that can used in community problem solving situations.The problem is broken down into stages in an effort to make the problem smaller and more manageable. The steps include:

- Identify the problem.
- Investigate the problem and provide related facts.
- Evaluate the data that has been collected.
- Brainstorm and list possible actions of what could happen.
- Predict possible outcomes from all solutions.
- Select the best action for the problem.
- Implement the action.
- Evaluate the action and determine whether it solved the problem.

Analytic Problem Solving involves the identification, diagnosis, and solving of problems. Often analytic problems are convergent and have only a single correct answer (Problems, 2002). The steps involved in analytic problem solving consist of the following steps:

- Identifying the problem.
- Describing the problem.
- Diagnosing a root cause for the problem.
- Verifying the root cause of the problem before applying the solution.
- Investigate the problem again with the solution in place to determine if the problem still exists or will reoccur. (Analytic).

Lavoie (1995) reports the following problem-solving teaching strategies that can be used with students to improve their problem solving techniques. These strategies model the breaking down of the problem into a series of steps which are then more easily solved. The Good Strategy User Model (Pressley, et al., 1989) allows the teacher to model problem solving steps while explaining the steps and providing feedback. The Strategy Intervention Model (Deshler and Schumaker, 1986) uses a teacher modeled think out loud method. The students follow by guided practice until they have mastered the problem solving steps involved. The Training Arithmetic Problem-Solving Skills (TAPS) method is used primarily for mathematical problem solving and stresses keeping successful problem solving skills while discarding ineffective skills and replacing them with more effective skills (Derry, 1989). The final problem solving method mentioned by Lavoie is the Explicit Prediction Teaching Strategy (EXPRTS) developed by Lavoie (1993). This method has the students use a hypothetico-predictive problem solving process. The students are taught explicitly when and how to apply different cognitive behaviors to hypothetical-predictive problems.

Problem solving models can also be applied through three different methods of reasoning (Problem Solving Models, 1998). These models include backward reasoning, forward reasoning, and opportunistic reasoning. Backward reasoning models work from the final goal back to the initial state. This is often the method used in solving science problems because the problem solver has a clear goal is in mind and does not necessarily wander aimlessly trying to find a solution to the problem. The "solution" in this case is the initial condition.

Forward reasoning works from the initial state towards the solution. This method is often used in solving problems where there seems to be a clear direction to head. Opportunistic reasoning occurs when the problem solver uses whichever direction seems most productive at that moment. This method is often used when solving trigonometry identities and might be misconstrued as a haphazard approach when in fact it is following a logical path for the current conditions of the problem.

Prior to beginning the problem solving process, Bedell and Lennox (1996) suggest seven guiding principles to follow to increase the probability of successful problem solving. These include:

- Problems are natural. Having problems is not bad and it does not imply weakness.
- Think before jumping to a solution. Do not act on the first solution that comes to mind.
- Most problems can be solved. Do not give up on the problem before trying to solve it.
- Take responsibility for problems. There is little control over problems attributed to an external agent.
- State what you can do, not what you can't do.
- Behavior must be legal and socially acceptable.
- Solutions must be within our power and ability. Solutions outside the individuals power and ability are doomed to failure.

Using these seven ideals can lead to greater success during the problem solving process. Effective problem solvers have several characteristics in common (Whimbey & Lochhead, 1995). These characteristics allow them to effective approach and begin solving problems. The characteristics are:

- Positive attitude. Good problem solvers believe that the problem can be solved using tried and true methods.
- Concern for accuracy. The problem and data need to be fully understood before beginning.
- Breaking the problem into parts. The problem is broken into smaller steps, each of which is easier to solve than the whole.
- Avoiding guessing. The problem is worked out in an effective order and in small steps. Guesses, flashes of intuition, and carelessness are to be avoided.
- Activeness in problem solving. Effective problem solvers put more effort into the problem solving process than do poor problem solvers. They try to provide themselves with a clearer picture of what is going on during the problem solving process.

The many problem solving models and strategies available to teachers make it possible to help students learn many techniques that will aid them in their problem solving processes. These techniques all have the students identify the problem, collect data, consider possible solutions, solve the problem, and evaluate the result and the solution process. Many or all of these techniques learned in math and science classes will be able to be applied by the student to problems they occur in later life either in their job, leisure, or home life. Problem solving techniques will certainly guide the students in their life-long learning processes.

Mathematics Problem Solving Scoring Guide

	Emerging	Developing	Proficient	Exemplary
Conceptual Understanding **Key Question**: *Does the student's interpretation of the problem using mathematical representations and procedures accurately reflect the important mathematics in the problem?*	1. Your mathematical representations of the problem were incorrect. 2. You used the wrong information in trying to solve the problem. 3. The mathematical procedures you used would not lead to a correct solution. 4. You used mathematical terminology incorrectly.	1. Your choice of forms to represent the problem was inefficient or inaccurate. 2. You used some but not all of the relevant information from the problem. 3. The mathematical procedures you used would lead to a partially correct solution. 4. You used mathematical terminology imprecisely.	1. Your choices of mathematical representations of the problem were appropriate. 2. You used all relevant information from the problem in your solution. 3. The mathematical procedures you chose would lead to a correct solution. 4. You used mathematical terminology correctly.	1. Your choice of mathematical representations helped clarify the problem's meaning. 2. You uncovered hidden or implied information not readily apparent. 3. You chose mathematical procedures that would lead to an elegant solution. 4. You used mathematical terminology precisely.
Strategies and Reasoning **Key Question**: *Is there evidence that the student proceeded from a plan, applied appropriate strategies, and followed a logical and verifiable process toward a solution?*	1. Your strategies were not appropriate for the problem. 2. You didn't seem to know where to begin. 3. Your reasoning did not support your work. 4. There was no apparent relationship between your representations and the task 5. There was no apparent logic to your solution. 6. Your approach to the problem would not lead to a correct solution.	1. You used an oversimplified approach to the problem. 2. You offered little or no explanation of your strategies. 3. Some of your representations accurately depicted aspects of the problem. 4. You sometimes made leaps in your logic that were hard to follow. 5. Your process led to a partially complete solution.	1. You chose appropriate, efficient strategies for solving the problem. 2. You justified each step of your work. 3. Your representation(s) fit the task. 4. The logic of your solution was apparent. 5. Your process would lead to a complete, correct solution of the problem.	1. You chose innovative and insightful strategies for solving the problem. 2. You proved that your solution was correct and that your approach was valid. 3. You provided examples and/or counterexamples to support your solution. 4. You used a sophisticated approach to solve the problem.

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	Emerging	Developing	Proficient	Exemplary
Computation & Execution **Key Question:** *Given the approach taken by the student, is the solution performed in an accurate and complete manner?*	1. Errors in computation were serious enough to flaw your solution. 2. Your mathematical representations were inaccurate. 3. You labeled incorrectly. 4. Your solution was incorrect. 5. You gave no evidence of how you arrived at your answer.	1. You made minor computational errors. 2. Your representations were essentially correct but not accurately or completely labeled. 3. Your inefficient choice of procedures impeded your success. 4. The evidence for your solution was inconsistent or unclear.	1. Your computations were essentially accurate. 2. All visual representations were complete and accurate. 3. Your solution was essentially correct. 4. Your work clearly supported your solution.	1. All aspects of your solution were completely accurate. 2. You used multiple representations for verifying your solution. 3. You showed multiple ways to compute your answer.
Communication **Key Question:** *Was I able to easily understand the student's thinking or did I have to make inferences and guesses about what they were trying to do?*	1. I couldn't follow your thinking. 2. Your explanation seemed to ramble. 3. You gave no explanation for your work. 4. You did not seem to have a sense of what your audience needed to know. 5. Your mathematical representations did not help clarify your thinking.	1. Your solution was hard to follow in places. 2. I had to make inferences about what you meant in places. 3. You weren't able to sustain your good beginning. 4. Your explanation was redundant in places. 5. Your mathematical representations were somewhat helpful in clarifying your thinking.	1. I understood what you did and why you did it. 2. Your solution was well organized and easy to follow. 3. Your solution flowed logically from one step to the next. 4. You used an effective format for communicating. 5. Your mathematical representations helped clarify your solution.	1. Your explanation was clear and concise. 2. You communicated concepts with precision. 3. Your mathematical representations expanded on your solution. 4. You gave an in-depth explanation of your reasoning.
Insights **Key Question:** *Does the student grasp the deeper structure of the problem and see how the process used to solve this problem connects it to other problems or "real-world" applications?*	1. You were unable to recognize patterns and relationships. 2. You found a solution and then stopped. 3. You found no connections to other disciplines or mathematical concepts.	1. You recognized some patterns and relationships. 2. You found multiple solutions but not all were correct. 3. Your solution hinted at a connection to an application or another area of mathematics.	1. You recognized important patterns and relationships in the problem. 2. You found multiple solutions using different interpretations of the problem. 3. You connected your solution process to other problems, areas of mathematics or applications.	1. You created a general rule or formula for solving related problems. 2. You related the underlying structure of the problem to other similar problems. 3. You noted possible sources of error or ambiguity in the problem. 4. Your connection to a real-life application was accurate and realistic.

Resources

Following are selected resources related to mathematical problem solving. Resources include online web resources and resources indexed by the ERIC system.

Web Resources:

Similar resources can be found by using the Google search engine and search terms such as math OR mathematics OR mathematical OR science OR scientific "problem solving" model OR models

Math Problem Solving Model

http://www.nwrel.org/msec/mpm/

The NWREL Mathematics Problem-Solving Model helps educators meet the challenges of teaching and assessing open-ended problem solving. The model includes a scoring guide for problem solving, open-ended tasks, and examples of student work for practice in scoring. Explore this site to learn more about the components of the model, teaching strategies for mathematics problem solving, and resources for teaching and learning

Information Problem Solving Research Models

http://www.bcps.org/offices/lis/models/elem.html

Toward a model of learning data representations (2001) http://act-r.psy.cmu.edu/publications/pubinfo?id=360

Downloadable full text document.

ERIC Citations:

Chambers, D. L. (1996). Direct modeling and invented procedures: Building on students' informal strategies. *Teaching Children Mathematics,* 3 (2). 92-95.

Presents examples of direct modeling and invented strategies of students for problem solving. Teachers are encouraged to capitalize on children's abilities to use these invented strategies and in time guide the student toward a standard, more efficient strategy. EJ 533 250.

Dooley, C. (1997). Problem-centered learning experiences: exploring past, present and future perspectives. *Roeper Review,* 19 (4), 192-195.

The problem-centered learning model for gifted students is described and applied to development of learning experiences that are organized around exploration of past, present, and future perspectives of trends, problems, events, and phenomena in the social sciences. Ways to use problem-centered learning in regular classrooms, special programs for gifted students, and self-contained gifted classes are noted. EJ 550 585.

Frederiksen, J. R., White, B. Y., & Gutwill, J. (1999). Dynamic mental models in learning science: The importance of constructing derivational linkages among models. *Journal of Research in Science Teaching,* 36 (7), 806-836.

Presents a theory of learning in science based on students deriving conceptual linkages among multiple models which represent physical phenomena at different levels of abstraction. Finds that high school students who were exposed to derivational links among three models for basic electricity performed better when solving both qualitative and quantitative problems on current and voltage. EJ 592 107.

Gravemeijer, K. (1997). Solving word problems: A case of modelling? Commentary. *Learning and Instruction,* 7 (4), 389-397.

It is argued that students who appear to ignore common sense and real-world knowledge in their approach to mathematics word problems in school are often behaving sensibly given the situation. The improvement of mathematics word problems for teaching will require changes in teacher beliefs and the use of modelling as an activity of organizing. EJ 553 114.

Greenes, C. (1995). Mathematics learning and knowing: A cognitive process. *Journal of Education,* 177 (1), 85-106.

Introduces the constructivist theory of mathematical learning, and illustrates its application in a curriculum focusing on fundamental concepts. How knowledge is constructed, how concepts mature, and what it means to learn and to know mathematics are presented. Pedagogical implications of adopting a constructivist perspective on knowledge and learning are considered, and two instructional models are presented. EJ 526 915.

Hatano, G. (1997). Cost and benefit of modelling activity. Commentary. *Learning and Instruction,* 7 (4), 383-387.

It is argued that mathematical modelling is important in teaching mathematics and that teachers have to create cultural support for it in the classroom. Posing an unsolvable problem for students may reveal the culture of the classroom as it exists, but its contribution to developing the needed culture that supports mathematical modelling appears limited. EJ 553 113.

Paul, C. (1995). The return of Matt Dillon. *Mathematics Teacher,* 88 (3), 192-195.

Uses probability models in problem-solving activities based on an episode of the television show "Gunsmoke." EJ 505 596.

Woodward, J. The role of models in secondary science instruction. *Remedial and Special Education (RASE),* 15 (2), 94-104.

This article reviews common curricular alternatives for addressing the science education needs of secondary students with learning disabilities. A curriculum revision method emphasizing models and context-rich problem solving is proposed as an alternative to the traditional uses of direct instruction, mnemonics, graphic organizers, and study guides. EJ 479 506.